SEM-2 (MATHEMATICS GENERAL)

ANUP RAY

Ex: Solve $(x^3y^3+x^2y^2+xy+1)y\,dx+(x^3y^3-x^2y^2-xy+1)x\,dy=0$

Ans: Compairing the given equation with the equation $M\,dx+N\,dy=0$ we get,

$M=y(x^3y^3+x^2y^2+xy+1)$ and $N=x(x^3y^3-x^2y^2-xy+1)$

$\therefore Mx-Ny=xy[x^3y^3+x^2y^2+xy+1-x^3y^3+x^2y^2+xy-1]$

$=2x^2y^2(xy+1)\neq 0$

$\therefore IF=\frac{1}{Mx-Ny}=\frac{1}{2x^2y^2(xy+1)}$

On multiplying the given equation by its I.F we get,

$$\frac{x^2y^2(xy+1)+1(xy+1)}{2x^2y^2(xy+1)}y\,dx+\frac{(xy+1)(x^2y^2-xy+1)-2xy(xy+1)}{2x^2y^2(xy+1)}x\,dy=0$$

or $\frac{x^2y^2+1}{2x^2y^2}y\,dx+\frac{x^2y^2-xy+1-2xy}{2x^2y^2}x\,dy=0$

or $(y\,dx+x\,dy)+\frac{y\,dx+x\,dy}{x^2y^2}-\frac{2x^2y}{x^2y^2}dy=0$

or $d(xy)+\frac{d(xy)}{(xy)^2}-2\frac{dy}{y}=0$

Integrating we get,

$xy-\frac{1}{xy}-2\log y=c$

Ex: Solve $y(x^2y^2+2)dx+x(2-2x^2y^2)dy=0$

Ans: Compairing the given equation with the equation $M\,dx+N\,dy=0$, we get,

$M=y(x^2y^2+2)$, $N=x(2-2x^2y^2)$

Now $Mx-Ny=xy(x^2y^2+2-2+2x^2y^2)$

$=3x^3y^3\neq 0$.

$\therefore$ I.F. $= \dfrac{1}{Mx - Ny} = \dfrac{1}{3x^3y^3}$

Multiplying the given equation by $\dfrac{1}{3x^3y^3}$ (I.F.) we get,

$$\left(\frac{1}{3x} + \frac{2}{3x^3y^2}\right)dx + \left(\frac{2}{3x^2y^3} - \frac{2}{3y}\right)dy = 0,$$

which is exact.

$$\int M\,dx \text{ (Taking } y \text{ as constant)} = \frac{1}{3}\int \frac{dx}{x} + \frac{2}{3y^2}\int \frac{dx}{x^3}$$

$$= \frac{1}{3}\log x - \frac{1}{3x^2y^2}.$$

$$\int N\,dy \text{ (Terms free from } x) = -\frac{2}{3}\int \frac{dy}{y}$$

$$= -\frac{2}{3}\log y.$$

$\therefore$ The general solution is

$$\frac{1}{3}\log x - \frac{1}{3x^2y^2} - \frac{2}{3}\log y = \frac{1}{3}\log c$$

$$\Rightarrow \log\frac{x}{cy^2} = \frac{1}{x^2y^2} \Rightarrow x = cy^2 e^{\frac{1}{x^2y^2}}$$

Ex: 3 Solve $(x^2y^2 + xy + 1)y\,dx + (x^2y^2 - xy + 1)x\,dy = 0$

Ans: Here $Mx - Ny = xy(x^2y^2 + xy + 1 - x^2y^2 + xy - 1)$

$$= 2x^2y^2 \neq 0$$

I.F. $= \dfrac{1}{2x^2y^2}$

$$\therefore \frac{1}{2}\left(y + \frac{1}{x} + \frac{1}{x^2y}\right)dx + \frac{1}{2}\left(x - \frac{1}{y} + \frac{1}{xy^2}\right)dy = 0$$

which is exact.

Now solve you as usual.

Ans: $xy - \dfrac{1}{xy} + \log\dfrac{x}{y} = c.$